SCORE: 23½ / 30 POINTS

1. NO CALCULATORS ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

MULTIPLE CHOICE. Consider the DEs

$\frac{5w+1}{\ln w - 3u} \frac{du}{dw} = 1$

SCORE: 0 / 3 PTS

[1] $(5r+1)\dfrac{dr}{d\theta} = 2\theta + 1$ [2] $x''y - y^2 = 2x$ [3] $(5w+1)du + (\ln w - 3u)dw = 0$

(where w is the independent variable)

Which of the DE above are linear ? Circle the correct answer below.

(a) none are linear (b) only [1] is linear (c) only [2] is linear (d) only [3] is linear

(e) only [1] & [2] are linear (f) only [1] & [3] are linear (g) only [2] & [3] are linear (h) all are linear

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Consider the autonomous DE $y' = (y-4)^2(1-y)$.

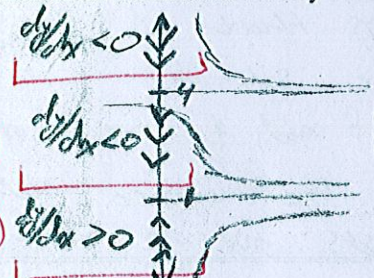
SCORE: 6 / 7 PTS

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

1

Zero at $y = 4, 1$

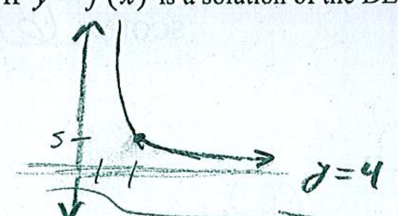
1 1 1



There is a semi-stable equilibrium solution at $y=4$ and a stable equilibrium solution at $y=1$

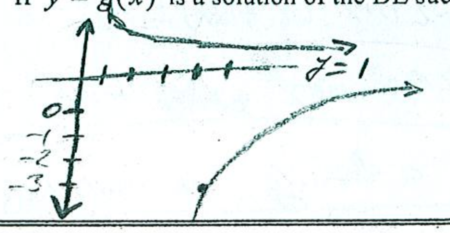
1

[b] If $y = f(x)$ is a solution of the DE such that $f(2) = 5$, what is $\lim\limits_{x\to\infty} f(x)$? HINT: Sketch a possible graph of $y = f(x)$.



$\lim\limits_{x\to\infty} f(x) =$ Does not exist because the equilibrium point at $y=4$ is semi-stable

[c] If $y = g(x)$ is a solution of the DE such that $g(5) = -3$, what is $\lim\limits_{x\to\infty} g(x)$?



$\lim\limits_{x\to\infty} g(x) = 1$ because y' is stable at the equilibrium point of $y=1$

1

Write a differential equation for the velocity $v(t)$ of a falling object if the air resistance is proportional to the square of the velocity. Assume that $v(t) > 0$ corresponds to the object moving upward, $v(t) < 0$ corresponds to the object moving downward. (NOTE: This is NOT the same problem as in the homework.)

SCORE: 3 / 3 PTS

let $v(t)$ = velocity of a falling object

air resistance
Object
gravity

$F_{total} = ma \Rightarrow F_{air} - F_g = m\frac{dv}{dt} \Rightarrow kv^2 - mg = m\frac{dv}{dt}$

$\boxed{\frac{dv}{dt} = \frac{kv^2 - mg}{m},\ k > 0}$

1 1 1

FILL IN THE BLANKS. SCORE: 1 / 3 PTS

[a] The order of the DE $y^{10} - y^{(7)}y^4 = (x^5 + y''')^6$ is 7. 1

[b] If $y = \sqrt{x+9}$ is a solution of the DE $y'' = f(x, y, y')$, the largest possible interval of definition is -9.

$y' = \frac{1}{2}(x+9)^{-1/2}$ $y'' = -\frac{1}{4}(x+9)^{-3/2}$

Consider the IVP $y' = 5x - 10y, \quad y(1) = -2$. SCORE: 5 / 5 PTS

Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

$y_{n+1} = y_n + h(f(x,y))$

$y(1.2) = -2 + .2(5(1) - 10(-2))$

$y(1.2) = -2 + .2(25)$ 2

$y(1.2) = -2 + 5$

$y(1.2) = 3$ 1

$y(1.4) = y(1.2) + h(f(x+h, y(1.2))$

$y(1.4) = 3 + .2(5(1.2) - 10(3))$

$y(1.4) = 3 + .2(6 - 30)$

$y(1.4) = 3 + .2(-24)$ 1

$y(1.4) = 3 - 4.8$

$\boxed{y(1.4) = -1.8}$ 1

1.2, 5, 60, 24, $\times .2$, 4.8

What does the Existence & Uniqueness Theorem tell you about the IVP $(\sin x)y' - y^{\frac{5}{2}} = 0, \quad y(\frac{\pi}{4}) = 0$? SCORE: $2\frac{1}{2}$ / 3 PTS

Justify your answer properly, but briefly.

The Exisitence and uniqueness theorem says that if f and f_y are continous everywhere then the DE has one solution.

$\sin x\, y' - y^{5/2} = 0$

$y' = \frac{y^{5/2}}{\sin x}$ $\frac{1}{2}$

$f = \frac{y^{5/2}}{\sin x}$, $f_y = \frac{5y^{3/2}}{2\sin x}$ not cont. 1

Because f and f_y are not continous everywhere, the existance and uniqueness theorem says nothing about it. 1

Consider the DE $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2$. SCORE: 6 / 6 PTS

[a] Is $y = Ax^3 + x^2 + Bx$ a family of solutions of the DE ?

$y = Ax^3 + x^2 + Bx$

$y' = 3Ax^2 + 2x + B$

$y'' = 6Ax + 2$

$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2$

1 1 1

$x^2(6Ax + 2) - 2x(3Ax^2 + 2x + B) + 2(Ax^3 + x^2 + Bx) = x^2$

$\cancel{6Ax^3} \cancel{+2x^2} \cancel{-6Ax^3} \cancel{-4x^2} \cancel{-2xB} + 2Ax^3 \cancel{+2x^2} \cancel{+2Bx} = x^2$

$2Ax^3 \neq x^2$ $1\frac{1}{2}$

$\boxed{\text{not a family of soltions}}$

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = -1, \quad y'(1) = 3$. $1\frac{1}{2}$

If the answer to [a] is "NO", skip this part.